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Dynamic characterization of bolted joints using FRF decoupling and optimization

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ABSTRACT

Mechanical connections play a significant role in predicting dynamic characteristics of assembled structures. Therefore, equivalent dynamic models for joints are needed. Due to the complexity of joints, it is difficult to describe joint dynamics with analytical models. Reliable models are generally obtained using experimental measurements. In this paper an experimental identification method based on FRF decoupling and optimization algorithm is proposed for modeling joints. In the method the FRFs of two substructures connected with a joint are measured, while the FRFs of the substructures are obtained numerically or experimentally. Then the joint properties are calculated in terms of translational, rotational and cross-coupling stiffness and damping values by using FRF decoupling. In order to eliminate the numerical errors associated with matrix inversion an optimization algorithm is used to update the joint values obtained from FRF decoupling. The validity of the proposed method is demonstrated with experimental studies with bolted joints.

1. Introduction

Many engineering structures are assembled from components by using a variety of connections, such as bolted, riveted, welded and bonded joints. For individual structures, modeling and response prediction methods have been well developed for decades. However, the prediction of dynamic characteristics of assembled systems is restricted by the capabilities to properly describe joints. Joints add damping to the structural system and decrease the overall stiffness; hence, changing the overall dynamic characteristics of the system [1]. Due to the effect of joints on the dynamic behavior of assembled structures, the importance of joint modeling or the identification of joint dynamic properties has become more and more significant. Reliable dynamic models for structural systems are based on the accurate identification of joint parameters.

Since dynamic modeling of joints analytically is very difficult and usually not so accurate, experimental methods are used as an alternative for establishing a mathematical model for a joint. Experimental methods can be classified as modal based methods and frequency response function (FRF) based methods. In the former class of methods, modal parameters obtained from modal testing are utilized in the identification of joint properties [2–4]. However, for structures including closely spaced modes or large modal damping, accurate modal parameters are not easily obtained. Moreover, due to the nature of the modal parameter extraction process, results inevitably contain errors to some degree. In order to overcome this

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shortcoming, FRF based methods have been proposed in the literature. In addition, Nobari [5] pointed out several advantages of the FRF based methods over modal based ones in his study. For instance, while FRF based identification methods eliminate the need for modal analysis, they also provide the flexibility of selecting proper data points for an identification process due to the large amount of information measured in the frequency domain.

The basic strategy in most of the FRF based joint identification methods is to use FRFs of individual substructures without joints and those of the assembled system (which is the structure with joints) to obtain information about the joint properties [6]. In the past decade, there were several researchers who focused on the identification of joint properties using FRF based methods. Although the formulations in such methods are usually quite straightforward, due to the inherent noise in measurements and sensitivities of the formulations to noise, accuracy of the identification results differs. The reasons for this sensitivity and ways of coping with them are investigated in several studies [6–21]. For example, Tsai and Chou [7] utilized the substructure FRF synthesis method in the formulation of the joint parameter identification. Wang and Liou [8] improved the work of Tsai and Chou [7]. They avoided inversion of matrices in their algorithm; hence, tried to reduce noise effect in the identification. Using a similar formulation, Hwang [9] identified the stiffness parameter of a connection by subtracting the inverted FRF matrices of the structure without joints from those of the structure with joints. Then, he improved the results using an averaging process to exclude highly sensitive regions. Wang and Chuang [10] developed a new identification algorithm to improve the results of Wang and Liou [8]. However, for the estimation of joint parameters, these methods require or use the measured FRFs related to the joint interface degrees of freedoms (DOFs) which are not easy to be obtained in real structures. Yang et al. [11] improved the methods [7,8] by eliminating the need for joint related FRFs and derived identification equations employing substructural synthesis. They modeled a joint in terms of translational and rotational stiffness values and used singular value decomposition to avoid noise effect. However, joint damping was not included in their work. Later, Hu et al. [12] used this approach in order to identify the dynamic stiffness matrix of bearing joint region. Yang and Park [13] also eliminated the necessity for joint related and rotational FRF measurements by developing an algorithm to estimate unmeasured FRFs from the measured FRFs. They proposed a method using subset FRF measurements for the identification of joint parameters, and in order to reduce the effect of identification errors they used weighting techniques.

Ren and Beards [14,15] developed a generalized coupling method taking into account the physical restrictions of the real structures, and identified joint parameters with this new method. However, they did not use stiff joints in order to avoid ill-conditioned matrices, and used weighting techniques for better accuracy [6]. By improving this method, Liu and Ewins [16] developed a new FRF coupling analysis method called generalized joint describing method. Čelič and Boltežar [17] also improved the method developed by Ren and Beards [1,6,7] by including the effects of rotational degrees of freedom (RDOFs). A more detailed and clear formulation of their method is given in their later work [18]. Silva et al. [19] presented uncoupling method and in order to avoid the direct use of data with experimental errors they regenerated the FRFs from a mathematical model using the modal parameters obtained from the experimental data. Later, this uncoupling technique is utilized for the dynamic characterization of joints [20,21]. The approach, in which the impedance uncoupling technique is reformulated and joints are identified without using joint related FRFs, is also similar to the one presented in [15]. This method has not been validated with an experimental study.

The FRF-based substructuring methods are so versatile that they were used in several studies for the joint identification of modular tools [22–24]. For instance, Özşahin et al. [22] implemented the elastic coupling method in a reverse manner, and they identified the contact parameters between spindle-holder and tool. In identification they used the measured FRFs of the spindle-holder–tool assembly (which can be regarded as a coupled structure), and the calculated or measured FRFs of the cutting tool, as well as the FRFs of the spindle-holder system (which can be regarded as substructures).

In this study an experimental identification method based on FRF decoupling and optimization algorithm is proposed for modeling structural joints. In the method proposed in an earlier study [25] FRFs of two substructures connected with a bolted joint are measured, while the FRFs of the substructures are obtained theoretically or experimentally. Then the joint properties are calculated in terms of translational, rotational and cross-coupling stiffness and damping values by using FRF decoupling. Hence, the joint is fully modeled. In order to eliminate the numerical errors associated with matrix inversion an optimization algorithm is proposed to update the joint values obtained from FRF decoupling [26,27]. Another advantage of the proposed method is that the joint related FRFs of the coupled system are not used in the identification formulations. The validity and application of the proposed method are demonstrated with several experimental studies by using beams connected with hexagonal bolts.

2. Theory

2.1. Identification of dynamic properties of joints using FRF decoupling

Frequency response function coupling is one of the most widely used methods in the literature in order to couple two structures elastically. Consider substructures A, B and their assembly (structure C) obtained by coupling them with a flexible element as shown in Fig. 1.

The coordinates j and k represent joint DOFs, while r and s are the ones that belong to the selected points of substructures A and B, respectively, excluding joint DOFs. Let us partition the receptance matrices of substructures A and B

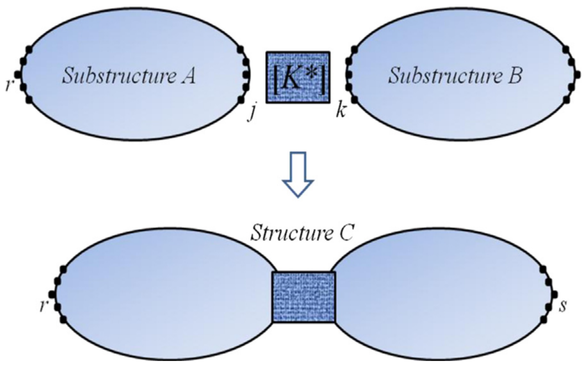

Fig. 1. Elastic coupling of two substructures.

and of the coupled structure C as follows:

$$[H_A(\omega)]=\begin{bmatrix}[H_{rr}(\omega)] & [H_{rj}(\omega)]\\ [H_{jr}(\omega)] & [H_{jj}(\omega)]\end{bmatrix},\quad [H_B(\omega)]=\begin{bmatrix}[H_{kk}(\omega)] & [H_{ks}(\omega)]\\ [H_{sk}(\omega)] & [H_{ss}(\omega)]\end{bmatrix},$$
$$[H_C(\omega)]=\begin{bmatrix}[H^C_{rr}(\omega)] & [H^C_{rs}(\omega)]\\ [H^C_{sr}(\omega)] & [H^C_{ss}(\omega)]\end{bmatrix} \tag{1}$$

Assuming no external forces and moments are acting on joints, the equations representing equilibrium of the forces and compatibility of displacements at connection DOFs can be written as

$$\{f_j\}+\{f_k\}=0 \tag{2}$$

$$[K^*]\{\{x_j\}-\{x_k\}\}=\{f_k\} \tag{3}$$

where $[K^*]$ is the complex stiffness matrix representing the joint dynamics and consists of the stiffness and damping elements that are to be identified.

Using Eqs. (1)–(3), receptance matrices of the assembly can be written as follows (the frequency dependency of the FRFs is not shown in the following equations for simplification):

$$[H^C_{rr}]=[H_{rr}]-[H_{rj}].[[H_{jj}]+[H_{kk}]+[K^*]^{-1}]^{-1}.[H_{jr}] \tag{4a}$$

$$[H^C_{rs}]=[H_{rj}].[[H_{jj}]+[H_{kk}]+[K^*]^{-1}]^{-1}.[H_{ks}] \tag{4b}$$

$$[H^C_{sr}]=[H_{sk}].[[H_{jj}]+[H_{kk}]+[K^*]^{-1}]^{-1}.[H_{jr}] \tag{4c}$$

$$[H^C_{ss}]=[H_{ss}]-[H_{sk}].[[H_{jj}]+[H_{kk}]+[K^*]^{-1}]^{-1}.[H_{ks}] \tag{4d}$$

By using the above equations, it is possible to decouple and thus calculate the complex stiffness matrix representing joint stiffness and damping as shown below

$$[K^*]=[[H_{jr}].[[H_{rr}]-[H^C_{rr}]]^{-1}.[H_{rj}]-[H_{jj}]-[H_{kk}]]^{-1} \tag{5a}$$

$$[K^*]=[[H_{ks}].[H^C_{rs}]^{-1}.[H_{rj}]-[H_{jj}]-[H_{kk}]]^{-1} \tag{5b}$$

$$[K^*]=[[H_{jr}].[H^C_{sr}]^{-1}.[H_{sk}]-[H_{jj}]-[H_{kk}]]^{-1} \tag{5c}$$

$$[K^*]=[[H_{ks}].[[H_{ss}]-[H^C_{ss}]]^{-1}.[H_{sk}]-[H_{jj}]-[H_{kk}]]^{-1} \tag{5d}$$

It can be easily seen that Eqs. (5a) and (5d) are symmetrical to each other. Similarly, Eqs. (5b) and (5c) are symmetrical to each other. If FRF matrices of the substructures and that of the coupled structure at any frequency are available, joint identification can be achieved by using any of the above equations, and, theoretically speaking, it does not make any difference which one of the equations are used in the extraction of the joint properties. However, as will be discussed in Sections 3 and 4, accuracy of each identification equation will differ depending on how the FRFs of the substructures are obtained. For example, in this study, for the structural system used in case studies, the most accurate results are obtained when Eq. (5a) is used, whereas from the experimental applicability point of view the most practical one is Eq. (5d).

Furthermore, again theoretically, the identification equations can be employed by using FRFs measured at any frequency. However, the effects of joint dynamics on system response are almost negligible at several frequencies, but very much pronounced at certain other frequencies. Therefore, it is not easy, if not impossible, to identify joint properties by using these equations at frequencies where the effects of joint dynamics on system response is almost negligible. At such frequencies equations will be very sensitive to noise in measured FRFs which is unavoidable in practical applications. Yet, if the equations are used at any frequency in a range where joint properties affect the system response considerably, the computations will be less sensitive to measurement noise. This point will be discussed further in the case study.

In the method proposed, in order to make the application more practical, instead of estimating the RDOF related FRFs of the coupled structure via experiments, only translational DOF (TDOF) related ones are measured and used in the identification equation. In this approach only one translational FRF at the tip point (point s) of the coupled structure (see Fig. 2) is used in the decoupling equation, Eq. (5d). Then, the matrix dimensions will be as follows:

$$\underset{(2\times 2)}{[K^*]} = [\underset{(2\times 1)}{[H_{ks}]}.[\underset{(1\times 1)}{[H_{ss}]} - \underset{(1\times 1)}{[H_{ss}^C]}]^{-1}.\underset{(1\times 2)}{[H_{sk}]} - \underset{(2\times 2)}{[H_{jj}]} - \underset{(2\times 2)}{[H_{kk}]}]^{-1} \tag{6}$$

Moreover, it is also observed in the experimental studies that, when the number of TDOF related FRFs used in the identification equation is increased, the identification yields more accurate results. When this is the case, the matrix dimensions given in Eq. (6) will change according to the number of measurement points around s. For example, when three TDOF measurements are taken from the tip point of the coupled structure, then there will be three nodes at s and the size of the matrices will be (3×3) for H_{ss} and H_{ss}^C, (2×3) for H_{ks} and (3×2) for H_{sk}; whereas the joint matrix and joint related DOFs for the substructures remain unchanged. Hence, in the identification process, the need for the estimation of RDOF related FRFs that belong to the coupled structure is eliminated, while keeping joint model unchanged with translational, rotational and cross-coupling elements.

After calculating the complex joint stiffness matrix, the stiffness and damping values representing the joint dynamics are obtained from the real and imaginary parts of the matrix elements, respectively. Note that, in this study joint damping is modeled with an equivalent viscous damping due to its ease of handling mathematically. According to Gaul and Lenz [28], the damping mechanism in joints is a much stronger function of displacement, in micro-slip regime, rather than velocity, so an equivalent viscous damping may seem to be non-phenomenological. However, joint dynamics affect the dynamics of the whole system usually only in a certain frequency range and the joint properties are identified around the sensitive modes that are most affected by the joint parameters. Therefore, viscous damping assumption still holds and the case studies reveal that using an equivalent viscous damping yields very good results.

2.2. Estimation of FRFs for RDOF and unmeasured coordinates

The formulation proposed in this study avoids the use of experimentally measured RDOF related FRFs for the coupled structure. However, we still need the RDOF related FRFs for the substructures.

The measurement of FRFs related to RDOFs is very difficult and requires special equipment [29–31]. Therefore, in experimental studies it is a general practice to obtain accurate and fast solutions for the estimation of unmeasured FRFs.

The RDOF related FRF estimation procedure is based on the well known technique using finite difference formulations. In this approach, the rotational information is derived from the translational measurements. Duarte and Ewins [32] reviewed the finite difference formulations to establish the best rotational data. In this study, second order central approximation technique suggested in reference [32] is used. In that approach three measurement points are required (points A, B and C in Fig. 3).

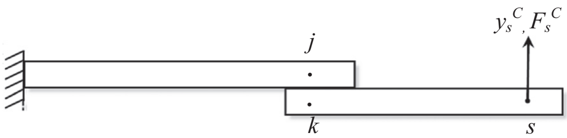

Fig. 2. Coupled structure with a bolted joint.

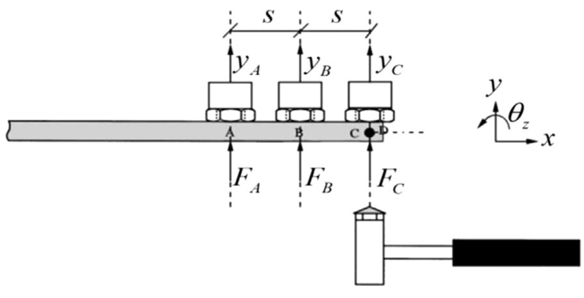

Fig. 3. Close-accelerometers method for RDOF measurements [32].

Rotational FRF at point B is calculated by using the second-order-central transformation matrix

$$[T_{2c}] = \frac{1}{2s}\begin{bmatrix} 0 & 2s & 0 \\ -1 & 0 & 1 \end{bmatrix} \tag{7}$$

as follows:

$$[H_{est}] = \begin{bmatrix} H_{yy} & H_{y\theta} \\ H_{\theta y} & H_{\theta\theta} \end{bmatrix} = [T_{2c}].[H_{meas}].[T_{2c}]^T \tag{8}$$

where $[H_{est}]$ represents the estimated FRFs in y and θ directions at point B, and $[H_{meas}]$ denotes the measured transational FRFs at points A–C, as shown below

$$[H_{meas}] = \begin{bmatrix} H_{AA} & H_{AB} & H_{AC} \\ H_{BA} & H_{BB} & H_{BC} \\ H_{CA} & H_{CB} & H_{CC} \end{bmatrix} \tag{9}$$

In the method proposed we need the full receptance matrix for the DOFs we are interested in. Numerically, all elements of an FRF matrix can be calculated easily. However, in real life applications, measuring all the elements of an FRF matrix experimentally is very time-consuming and expensive; besides, it may not be possible for all cases. Usually, only one column of an FRF matrix can be obtained by exciting the structure from a single point and measuring responses at all the points we are interested in. Then, the incomplete data (in any of the FRF matrices we need to use) can be obtained by using FRF synthesis [33] after extracting modal parameters by modal testing, using only one column of the FRF matrix.

2.3. Optimization and joint parameter updating

In this study it is intended to obtain constant joint parameters representing the joint stiffness and joint damping in the whole frequency range of interest. In our previous study, stiffness and damping values are identified at every frequency in the mode which is sensitive to the joint dynamics and then the averages of these identified values are taken as the resultant values [25]. However, in real life applications due to the measurement errors and numerical errors associated with matrix inversion, the variation of identified values with frequency may be considerable and it may not be easy to decide on the best frequency range to be used even in the mode which is sensitive to joint dynamics for identifying the joint properties correctly. According to Young et al. [34], joint stiffness matrix may acquire inertial properties during decomposition process which results in varying properties for the joint. Then, they proposed a solution for this problem by taking some fit values from the identification curve and using these values to redefine the joint stiffness matrix in a dynamic manner. However, our aim is to obtain constant joint properties so that the identified joint properties can be used in FE analysis to predict the dynamic response of the assembly.

In the present study, an optimization algorithm is developed using “fminunc” command of MATLAB to optimize the joint properties such that regenerated and measured FRFs for the bolted assembly match at all the modes in the frequency range of interest. The joint parameters identified by employing the method proposed in our earlier work [25] are used as initial estimates. Then this algorithm is used to update the joint parameters by minimizing the sum of the difference between the squares of the actual and regenerated receptance amplitudes calculated at each frequency. The key point for the success of the optimization process lies in starting with a good initial estimate. In fact, one can obtain a set of joint parameters starting with some arbitrary initial estimates and identify joint parameters which may yield the correct FRF curve used in the optimization process. However, when these values are used in a new assembly, the correct FRFs cannot be obtained due to lacking of a physical basis of the joint dynamic properties. Hence, in order to have a correct dynamic model of a joint through optimization, it is concluded that, initial estimates used in optimization process for the joint parameters should be close to the actual values, which can be obtained by employing the identification equations given in Section 2.1. The application of the proposed method is illustrated and then verified with several experimental studies conducted by using two beams connected with hexagonal bolts.

3. Simulation studies

In this section a case study is given to verify and illustrate the application of the method suggested. In the case study, two identical beams, substructure A having fixed–free boundary condition and substructure B having free–free boundary condition, are coupled elastically with a bolted joint as shown in Fig. 4. Each substructure is modeled with beam elements using finite element method (FEM). Displacements at each node are modeled with one translational and one rotational DOF. Then, FRFs of the substructures are calculated numerically.

In order to simulate experimental FRFs for coupled bolted structure, the FRFs calculated by using a mathematically known bolt model are polluted with 5% noise. The following data are used for the beams:

Beam length: $L = 0.3$ m; modulus of elasticity: $E = 2.07 \times 10^{11}$ N/m^2; moment of inertia of the cross-sectional area: $I = 1.0667 \times 10^{-9}$ m^4; mass per unit length of the beams: $m = 1.5094$ kg/m; damping of the beams: structural damping with a loss factor of 0.05.

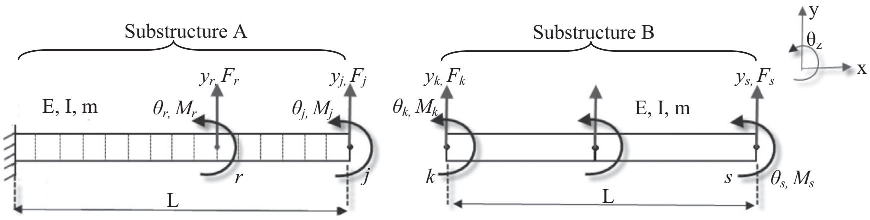

Fig. 4. Coupling of two beams with bolted joints.

Table 1
Joint parameters identified by using different decoupling equations (Eqs. (5a), (5b) and (5d)).

	k_{Fy} [N/m]	$k_{F\theta}$ [N/rad]	$k_{M\theta}$ [N m/rad]	c_{Fy} [N s/m]	$c_{F\theta}$ [N s/rad]	$c_{M\theta}$ [N m s/rad]
Actual values	10^6	10^3	10^4	25	5	8
Identified values (5a)	1.01×10^6	1.11×10^3	9.03×10^3	25.2	4.83	5.60
Error (%)	*1.3*	*11*	*−9.7*	*0.8*	−3.4	−30
Identified values (5b)	9.91×10^5	178	8.60×10^3	43.2	3.52	8.19
Error (%)	*−1*	*−82*	*−14*	72	−30	3
Identified values (5d)	9.02×10^6	9.66×10^2	2.38×10^4	76.3	3.36	13.5
Error (%)	*10*	*−4*	*138*	205	−32	68

Here, it is assumed that the bolted joint can be modeled with a complex stiffness matrix with translational and rotational stiffnesses, and translational and rotational viscous damping elements, as shown below:

$$[K^*]=\begin{bmatrix} k_{Fy}+j\omega c_{Fy} & k_{F\theta}+j\omega c_{F\theta} \\ k_{My}+j\omega c_{My} & k_{M\theta}+j\omega c_{M\theta} \end{bmatrix} \tag{10}$$

The following data are used for the joint:

Translational stiffness: $k_{Fy}=10^6$ N/m; rotational stiffness: $k_{M\theta}=10^3$ N m/rad; cross-coupling term between the translational and rotational stiffness: $k_{F\theta}=10^4$ N/rad and $k_{My}=10^4$ N m/m; translational damping: $c_{Fy\theta}=25$ N s/m; rotational damping, $c_{M\theta}=5$ N m s/rad; cross-coupling term between the translational and rotational damping: $c_{F\theta}=8$ N s/rad and $c_{My}=8$ N m s/m.

It is observed that exact values are calculated for joint stiffness and damping when the exact FRF values are used for the coupled system. However, when polluted FRFs are used, although very accurate values are obtained at some frequencies, the results are deteriorated and deviations from the actual values are drastic at some other frequencies. It is also observed that the performance of each identification equation differs considerably. In order to compare the performances of four decoupling equations (Eqs. (5a)–(5d)), the joint identification is performed using each equation by following the procedure described in Section 2.1 (which is the method proposed in [25]), and the accuracy of the joint parameters identified from each equation is compared. As it can be seen from Table 1 the best performance is obtained from Eq. (5a). The identified joint stiffness and damping values for the best performance case are given in Figs. 5 and 6, respectively.

In the previous study [25], it was shown that the FRF decoupling method works well in the frequency range where the coupled system FRFs are most sensitive to joint dynamics. Then it was concluded that in the identification of the joint properties, FRFs measured at frequencies where connection stiffness has less effect on the response of the coupled system should be avoided; instead, any set of FRFs, measured at any frequency in the frequency range at which connection dynamics affect coupled system dynamics the most, can be used and accurate identification can be made. Since it is not known in advance at which mode the joint dynamics will affect the coupled system dynamics considerably, it is the best to identify joint properties in a range of frequency and take the average of the values in a region where deviations from a constant value is minimum. In this case study, the following ranges are used in the identification of the joint properties: 200–400 Hz for the translational joint properties, 15–300 Hz for the rotational joint properties and 150–310 Hz for the cross-coupling joint properties. Note that, the damping properties prone to noise much more than the stiffness properties, since their effects on the coupled system dynamics is much less than those of joint stiffness values. Hence, for the damping terms, the frequency ranges used for identification of stiffness values are employed. The average values of the identification results in these ranges are given in Table 1. It can be seen that the best identified values have up to 30% deviation from the true values, which is due to the magnification of measurement errors in matrix inversion operations. Furthermore, when the mode shapes given in Fig. 7 are examined along with the sensitivity of the coupled system FRFs to the joint parameters, it is seen that the translational joint stiffness is very effective at the third and fourth modes and rotational joint stiffness has little effect at the second mode, while the cross-coupling joint stiffness has negligible effect on the receptances of the coupled system.

Then, by using the identified joint parameters, FRFs of the assembled system are regenerated and they are compared with the actual FRFs in Fig. 8. It can be seen from the comparison that the FRFs regenerated by using the identified joint

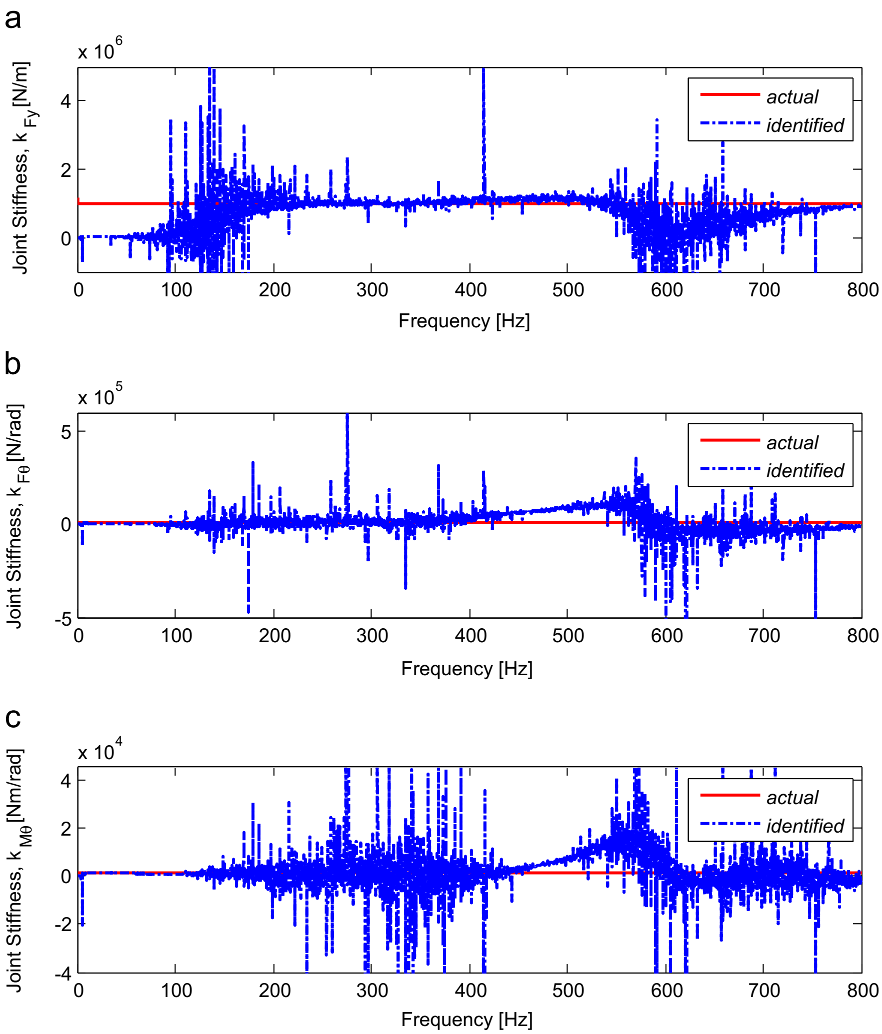

Fig. 5. Identification of joint stiffness: (a) translational joint stiffness; (b) cross-coupling joint stiffness; and (c) rotational joint stiffness.

parameters perfectly match with the actual FRFs. This case study reveals that having large errors in the identified values of rotational and especially cross-coupling joint stiffness values will not deteriorate the mathematical model for the joint, as long as translational stiffness of the joint is accurately identified. Main reason is that the effect of these parameters on system dynamics is not so significant. That is, when the effect of some parameters on system dynamics is insignificant, it is difficult to identify the accurate values of these parameters, and due to this exact reason, having inaccurate values for these parameters do not deteriorate the mathematical model for the joint. From the case study using simulated experimental data it can be concluded that the identification method is successful and is promising to be applied to real systems for the identification of joint dynamics.

4. Experimental verification

In this section two experimental studies are presented to verify the method suggested and to illustrate the accuracy and applicability of the method in real life applications. In these experiments, the joints connecting two steel beams with M10x35 and M8x35 bolts are identified and the models obtained for the connections are used in predicting FRFs either at the points which are not used in identification, or for new assemblies. In all experiments, a cantilever steel beam is used as substructure A, and it is connected to a free–free steel beam (substructure B) with the same cross section and material.

In the first experimental study, the joint obtained connecting two beams with M10x35 bolt is identified and the joint model is used to predict the FRFs of other points on the assembled system which are not used in identification computations. The predicted FRFs are compared with experimentally measured ones. In the second experimental study, a similar joint with M8x35 bolt is identified and the dynamic model of the joint is used to predict FRFs of a different coupled

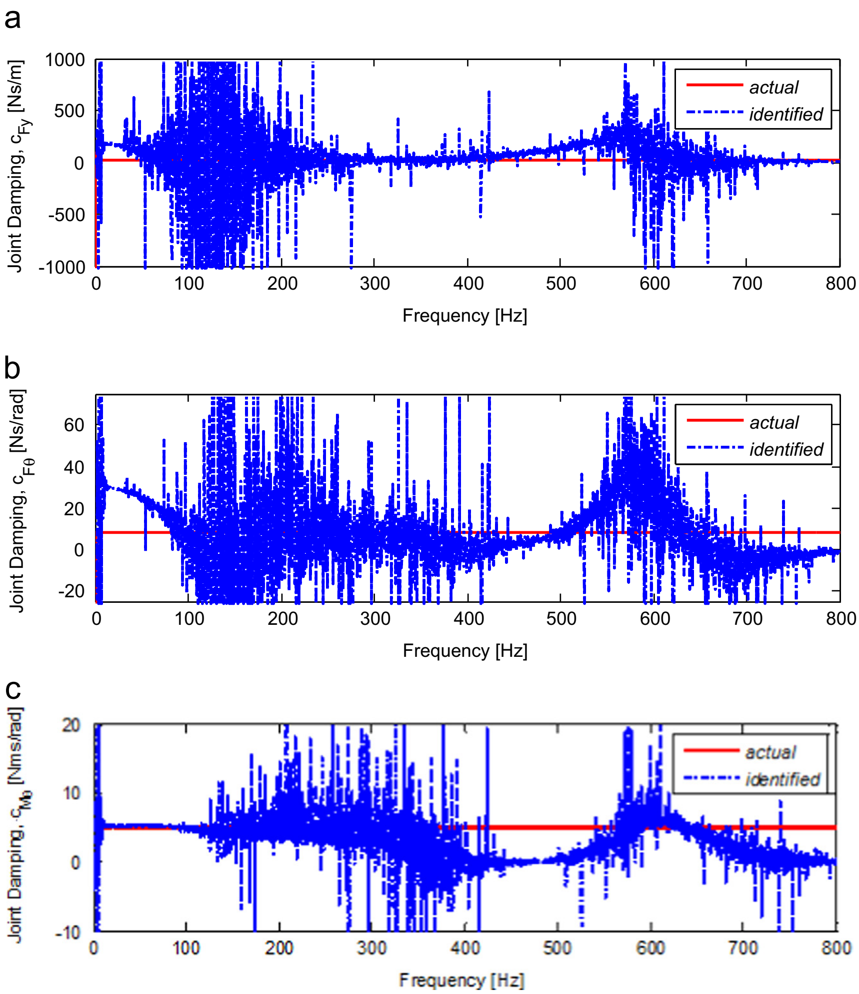

Fig. 6. Identification of joint damping: (a) translational joint damping; (b) cross-coupling joint damping; and (c) rotational joint damping.

structure obtained with the same bolt but with a longer cantilever beam. The predicted FRFs are again compared with experimentally measured ones.

In the identification processes, the simplified approach based on Eq. (5d) is used since from the experimental applicability point of view among the decoupling equations the most practical one is Eq. (5d). As demonstrated in Section 3 the most accurate results are obtained when Eq. (5a) is used. But it requires the measurements of cross-coupling FRFs including RDOF information between joint and non-joint coordinates, which is experimentally very difficult. Similarly, Eqs. (5b) and (5c) involve the cross-coupling FRFs which are difficult to measure experimentally. On the other hand, when Eq. (5d) is used, only the translational and rotational FRFs at the joint coordinate of substructure A which has fixed–free boundary conditions are required to be measured. Then FRFs of substructure B which has free–free boundary conditions are obtained theoretically from the FE model which is updated by using the modal parameters identified from experimental modal analysis of the beam. Hence, it can be concluded that for this system the most practical decoupling equation is the last one (Eq. (5d)).

In the experiments, LMS modal test system, PCB impact hammer, PCB miniature sensors are used. Frequency resolution is selected as 0.25 Hz and the frequency range is set to 512 Hz using soft tip of the impact hammer. For the free–free tests of substructure B, the frequency range is set to 1024 Hz using hard tip of the impact hammer.

4.1. *Experimental study I: verification of the method*

In this case study A2-70 M10x35 hexagon head bolt is used and the tightening torque is set to 30 N m. The joint obtained connecting two beams with the bolt is identified and this joint model is used to predict the FRFs of three different points on the assembled system; two of these points are being the ones which are not used in identification computations. The predicted FRFs are compared with experimentally measured ones.

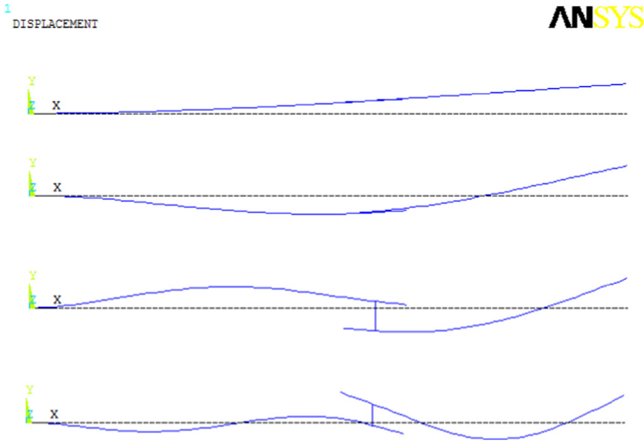

Fig. 7. Mode shapes of the coupled structure.

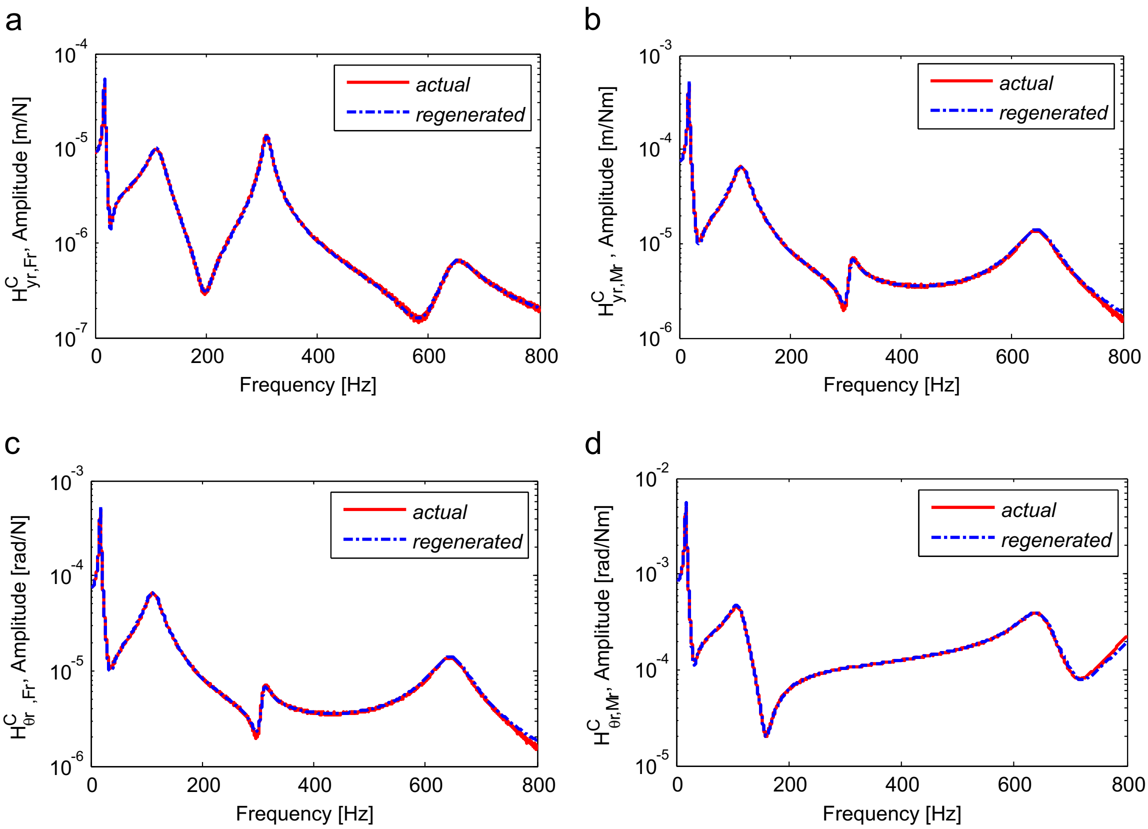

Fig. 8. Regenerated FRFs of the coupled structure by using identified joint properties: (a) $H^C_{yr,Fr}$; (b) $H^C_{yr,Mr}$; (c) $H^C_{\theta r,Fr}$; and (d) $H^C_{\theta r,Mr}$.

The test set-up is shown in Fig. 9. The dimensions of two stainless steel beams are as follows: length of substructure A, L_A=0.3 m; length of substructure B, L_B=0.335 m; width of the beams: 0.015 m, height of the beams: 0.010 m.

As explained in Section 4, in the experimental identification of the bolted joint, FRFs of substructure A which has fixed–free boundary conditions are directly measured. However, the FRFs of substructure B which has free–free boundary conditions are obtained theoretically by FE modeling, but using the structural parameters identified from experimental modal analysis of the beam. Note that this approach would yield accurate results for substructure B which has free boundary conditions, and a similar approach would not work for substructure A which has a clamped end of which flexibility cannot be numerically modeled. In order to estimate the RDOF related FRFs of substructure A, three accelerometer measurements are taken by exciting the system at point 1_a. The accelerometers are located with spacing, s (0.015 m) as shown in Fig. 10(a). After completing the FRF measurements, system identification is performed using the LMS modal analysis software [35], and

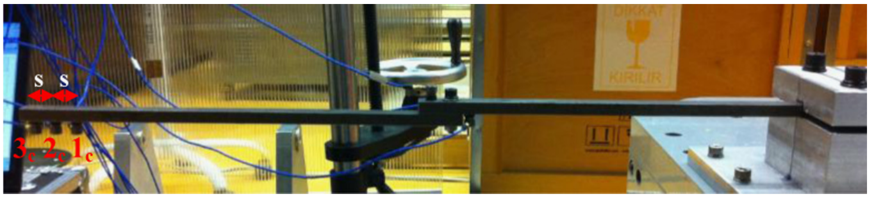

Fig. 9. Coupled structure with a bolt connection.

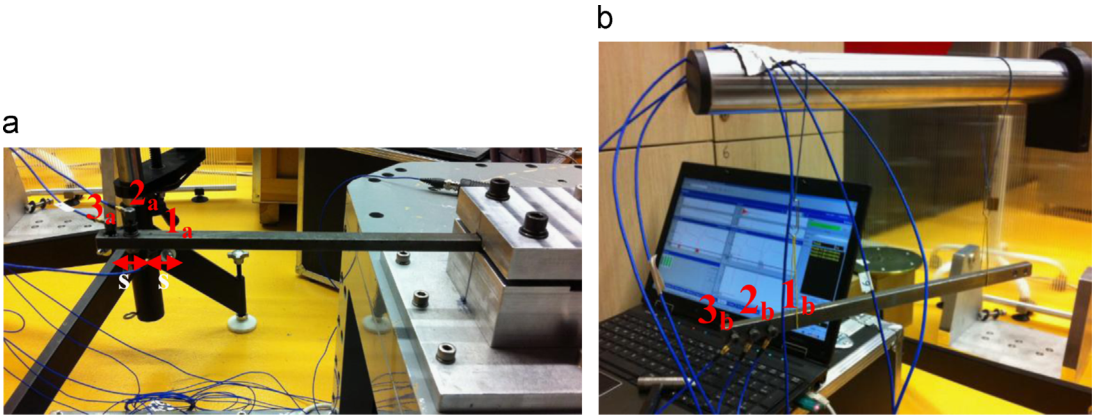

Fig. 10. (a) Close accelerometers method for substructure A and (b) testing of substructure B.

required parameters for the FRF synthesis are obtained in terms of upper and lower residuals, modal vectors, natural frequencies and damping ratios. Then, 3×3 translational FRF matrix representing the translational FRFs at the tree tip points of substructure A is obtained. Then, using the second order finite difference formula, rotational FRFs are estimated at point 2_a.

In the testing of substructure B, it is suspended with elastic cords (Fig. 10(b)). After exciting substructure B from the tip point, the tip point FRFs are experimentally obtained and the modal parameters are identified. Then by using model updating the updated parameters for substructure B are obtained as follows: elastic modulus $E=1.92\times10^{11}$ N/m^2 and density $\rho=7604$ kg/m^3. Finally, by using these values and the damping ratio identified for the first elastic mode (0.0005) the updated FE model of the free–free beam is obtained and the required FRFs of the substructure B ($[H_{kk}]$, $[H_{ks}]$, $[H_{sk}]$ and $[H_{ss}]$), including those related with RDOFs, are calculated accurately. Thus the problem of measuring cross-coupling FRFs including RDOFs is avoided.

In this experimental study the properties of the bolted joint are extracted by using Eq. (6), in which only one of the translational direct point FRF measured at the tip point 2_c of the coupled structure (Fig. 9) is used. The identified joint stiffness and damping values at each frequency up to 500 Hz are shown in Fig. 11.

It is seen that the joint properties are changing with frequency and the best frequency range to determine the joint properties is not obvious. When the averages of the identified values at a mode of the coupled structure are taken as the mathematical model of the joint and used in regenerating FRFs of the coupled structure, they are found to be exactly the same as the measured FRFs at that mode, but showing differences at other modes, indicating that these values cannot represent the bolted joint accurately at every mode. Yet, it was suggested in our previous study to use the values identified at the frequencies where FRFs are sensitive to joint dynamics, and take the average of them for the best possible identification. For this, first a sensitivity analysis is to be performed (Fig. 12).

From the sensitivity analysis, it is seen that, translational and cross-coupling joint stiffness are effective at the third mode, while rotational stiffness is effective at both second and third modes. Therefore, the values of translational and cross-coupling joint stiffness and damping identified at the third mode, and rotational parameters identified at the second mode give the best possible values. Then, these values are used as initial estimates. In the second stage of the joint identification, the initial estimates are updated with the optimization algorithm suggested in this study. The updated joint parameters are given in Table 2 along with initial estimates used. Note that initial estimates would represent the identified joint parameters if the updating was not performed.

By using the updated joint parameters, receptances of the assembled system are regenerated and they are compared with the measured receptances in Fig. 13. It can be seen from the comparison that the receptances regenerated by using the updated joint parameters perfectly match with the measured FRFs. It is interesting to note that while some joint parameters do not change much after updating, some change significantly (especially some cross terms), but the overall effect of updating is very significant at some modes as can be seen from Fig. 13.

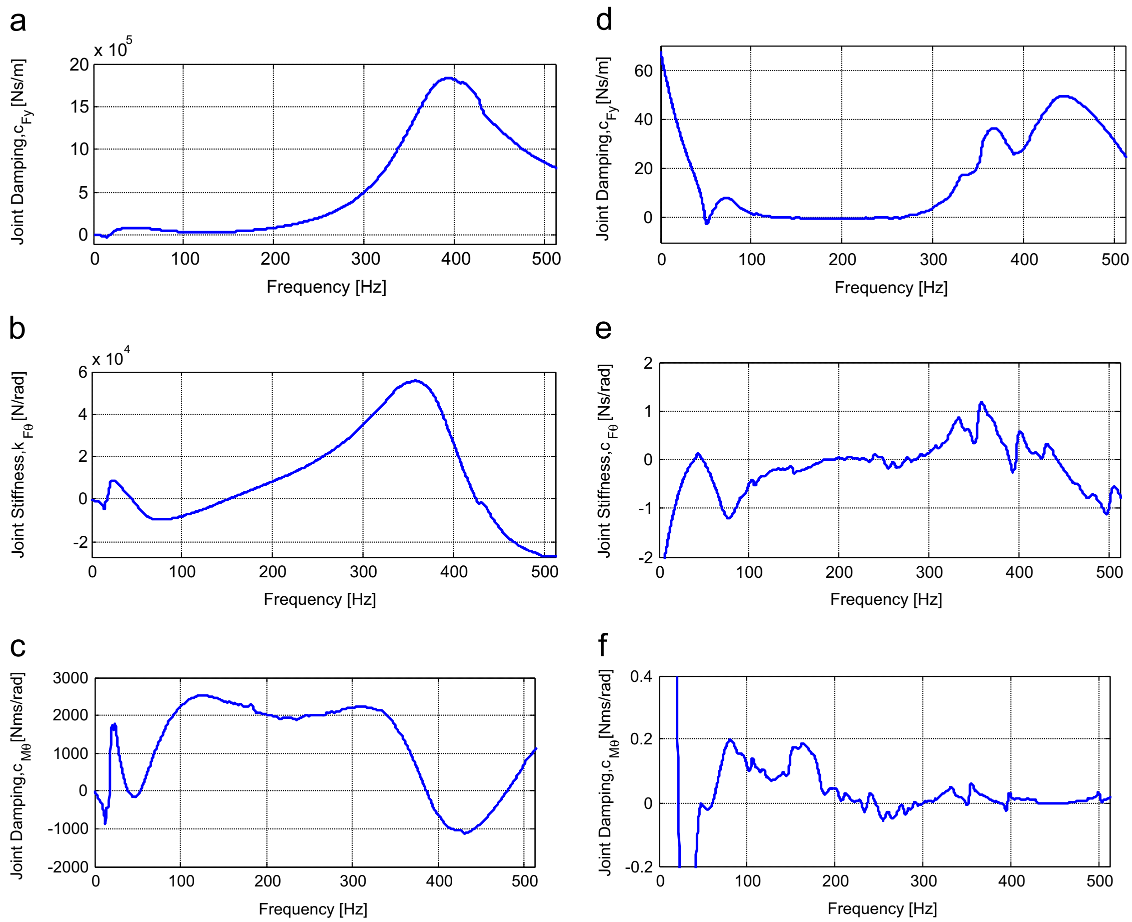

Fig. 11. Identification of joint stiffness and joint damping: (a) translational joint stiffness; (b) cross-coupling joint stiffness; (c) rotational joint stiffness; (d) translational joint damping; (e) cross-coupling joint damping; and (f) rotational joint damping.

Finally, by using the joint parameters identified (updated ones), the FRFs of the coupled structure at point 1_c and point 3_c (see Fig. 9), which are not used in the identification of joint properties, are obtained. It can be seen from Fig. 14 that the receptances calculated by using the updated joint parameters perfectly match with the measured FRFs. Hence, it can be concluded that, the joint properties are identified very accurately.

4.2. Experimental study II: verification of the identified mathematical model for a joint

In this experimental study, the dynamic properties of the joint obtained by using M8x35mm hexagon head bolt in connecting the two beams described above are determined. Then, the identified joint parameters are employed to calculate the FRFs of a different coupled structure constructed with the same bolt, but a longer cantilever beam having the same material and cross sectional dimensions. The predicted FRFs are compared with the measured ones.

First, again starting with initial estimates obtained from the identified joint properties as above, the updated joint parameters are obtained using optimization. In the identification of the bolted joint parameters, only one of the translational FRFs measured at the tip point (point 2_c) of the coupled structure is used. The results obtained are given in Table 3. The initially identified values (which are used as initial estimates) are also shown in the same table. By using the updated joint parameters, receptances of the assembled system are regenerated and they are compared with the measured ones in Fig. 15. It can be seen from the comparison that the receptances regenerated by using the updated joint parameters perfectly match with the measured FRFs.

After obtaining the joint parameters of the connection with M8 bolt, a different verification study is performed. In this verification study, a new substructure A with a length of 0.35 m (longer than the previous one) is used as shown in Fig. 16. By using the measured FRFs of the new substructure A, but joint parameters identified from the previous system (with a shorter substructure A), receptance of the new assembled system at the tip point is calculated, and it is compared with the measured receptance in Fig. 16. It can be seen from the comparison that the receptances predicted by using the joint parameters identified from a different assembly perfectly match with the measured FRFs. Then it can be concluded that,

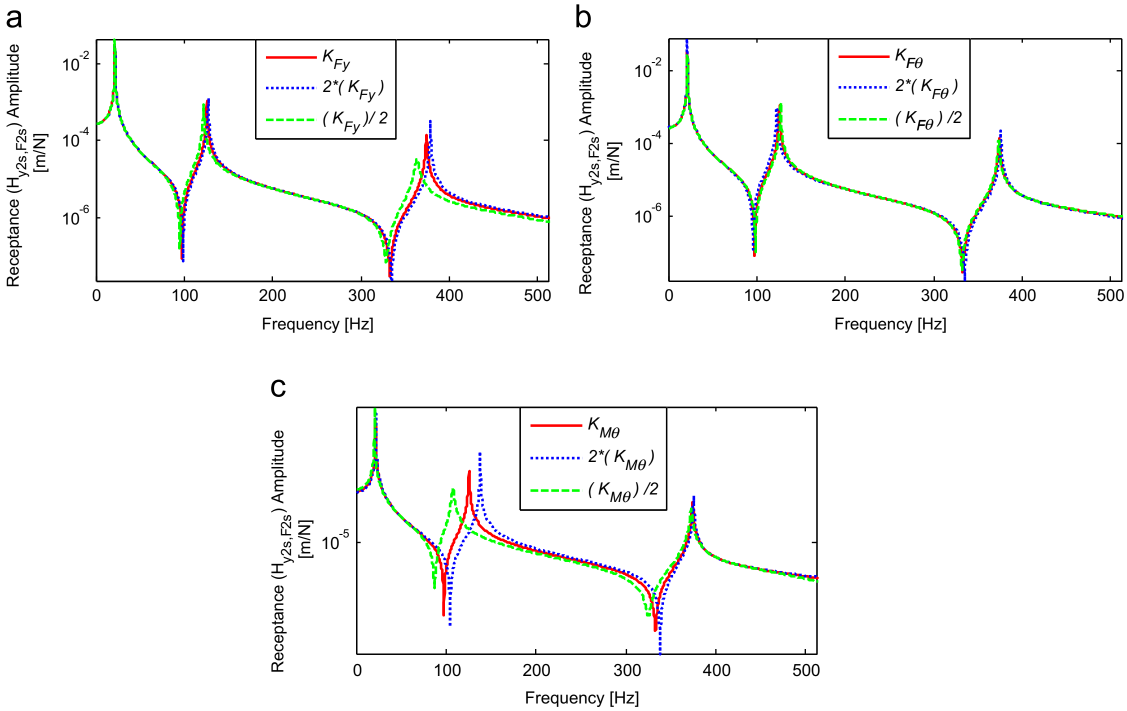

Fig. 12. Sensitivity of the receptance of the coupled structure to (a) translational joint stiffness, (b) cross-coupling joint stiffness, and (c) rotational joint stiffness.

Table 2
Joint parameters of the M10x35 bolted connection.

	k_{Fy} [N/m]	$k_{F\theta}$ [N/rad]	$k_{M\theta}$ [N m/rad]	c_{Fy} [N s/m]	$c_{F\theta}$ [N s/rad]	$c_{M\theta}$ [N m s/rad]
Initial estimates	1.71×10^6	5.12×10^4	2.5×10^3	34.72	0.646	0.073
Updated	2.02×10^6	2.47×10^4	2.8×10^3	34.41	0.635	0.077

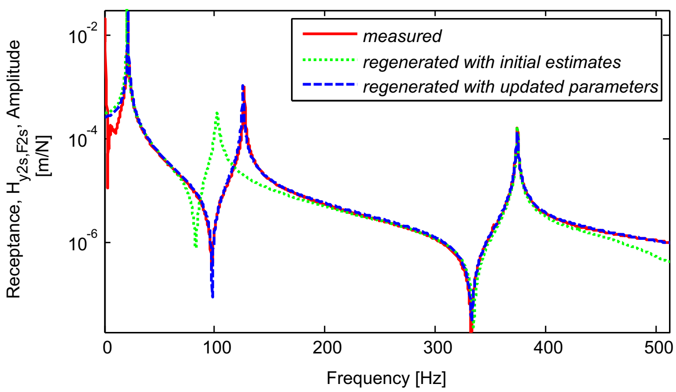

Fig. 13. Regenerated FRF, $H_{y2s,F2s}$, of the coupled structure using updated joint properties.

once the joint properties are identified, it can be used for another structure having the same connection (the same material and cross sectional dimensions for the beams, and the same bolt).

5. Discussions and conclusions

In this paper an identification method for mathematical modeling of a structural joint for dynamic analysis is proposed. The method is based on FRF decoupling using measured FRFs and on an optimization algorithm to update the identified joint parameters. In the FRF decoupling stage, FRFs of a structure composed of two separate structures (substructures)

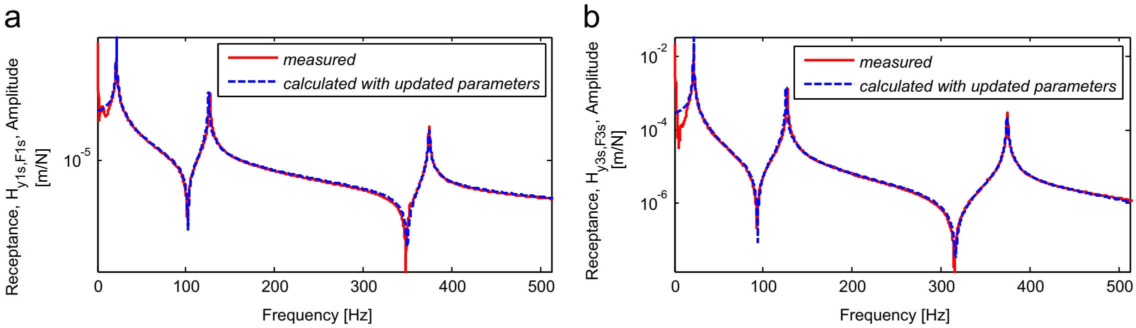

Fig. 14. Calculated FRFs of the coupled structure using updated joint properties: (a) $H_{y1s.F1s}$ and (b) $H_{y3s.F3s}$.

Table 3
Joint parameters of the M8x35 bolted connection.

	k_{Fy} [N/m]	$k_{F\theta}$ [N/rad]	$k_{M\theta}$ [N m/rad]	c_{Fy} [N s/m]	$c_{F\theta}$ [N s/rad]	$c_{M\theta}$ [N m s/rad]
Initial estimates	1.83×10^6	5.53×10^4	2.9×10^3	42.2	0.848	0.139
Updated	2.14×10^6	2.66×10^4	3.2×10^3	41.2	0.845	0.119

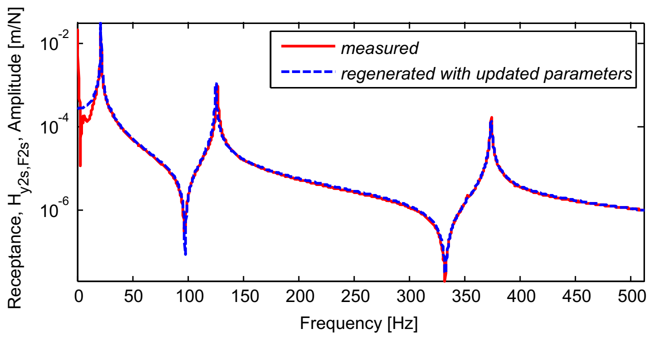

Fig. 15. Regenerated FRF, $H_{y2s.F2s}$, of the coupled structure using updated joint properties.

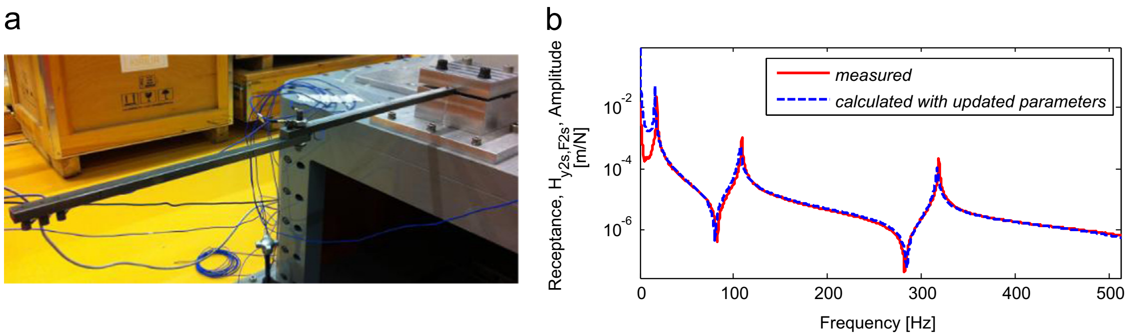

Fig. 16. (a) New assembly with same M8 bolt and (b) calculated FRF, $H_{y2s.F2s}$, of the new assembly using updated joint properties.

connected with a joint to be identified are measured, as well as those of the two substructures. The joint properties expressed in terms of rotational and translational stiffness and damping values are identified by using FRF based substructure decoupling equations. Four equations are presented for joint identification, each using a different set of FRFs but yielding the same joint properties. These four equations are, in pairs, symmetrical of each other; therefore, if we have symmetrical boundary conditions for the coupled system, effectively we have two equations. But, if the boundary conditions are not symmetric, then practically we have four different equations. If we could have the exact values of the FRFs for substructures and for coupled structure at any frequency, any of the joint identification equations would give exactly the same (and correct) results. However, due to using different sets of FRFs in each equation and due to having different measurement noise levels in each type of FRF, it is expected to have different performances for each equation. It is very well known that in FRF decoupling applications, the results are prone to large errors, due to the sensitivity of the equations to measurement errors. The most accurate results are obtained with Eq. (5a) for the system used in the case studies in this work (a cantilever beam bolted to a free–free beam). However, this formulation requires the measurement of cross-coupling FRFs including RDOF information between joint and non-joint coordinates, which is experimentally very difficult. So from the experimental applicability point of view, among the decoupling equations the most practical one is found to be Eq. (5d), for the system used in the experimental studies in this work. In the method proposed in this study, in order to obtain RDOF related FRFs of one of the substructures (fixed–free beam), finite difference method is employed using the three translational FRFs at the tip point. For the free–free substructure, first, measured translational FRFs are employed to identify some parameters of the substructure more accurately, then the FRFs are calculated from the FE model of the substructure. In a way, model updating is applied in order to improve the FE model of this substructure. If the decoupling equations are to be used directly for the identification, the measured values of RDOF related FRFs for the coupled structure are required as well. Although they can be predicted from translational FRFs of the points of concern and those of neighbor points, in order to make the method more practical for real applications, it is avoided to use RDOF related FRFs for the coupled structure, and it is suggested to measure and use only TDOF related ones. In the experimental applications given in this study only one translational FRF at the tip point of the coupled structure is used in the decoupling equation, eliminating the need for estimating RDOF related FRFs from TDOF related FRFs for the coupled structure, and it is demonstrated that this approach yields very accurate results. It is also observed that the accuracy can even be improved by using more than one point at the free end of the cantilever beam.

In the method proposed, the joint identification can be made without using the FRFs at the joint of the coupled system, which are more difficult to measure in practical applications. This is one of the advantages of the method suggested.

Although only the FRFs for selected points of the substructures are used in the formulation, the whole FRF matrix for these points is required in the computations. However, in practical applications it is a common practice to measure one column of the matrix. The unmeasured FRF values are calculated by using modal identification and FRF synthesis.

The joint parameters identified at different frequencies are usually found to be significantly different than each other due to the measurement errors and also due to the numerical errors associated with matrix inversion involved in computations. Therefore, in the identification of joint parameters it is proposed to take the averages of the values identified at frequencies in the mode which is most sensitive to these parameters. In experimental studies it is observed that the joint properties identified by taking averages are pretty accurate, but still need to be improved. Significant improvement is obtained by updating the joint parameters with an optimization algorithm suggested. Using optimization to match calculated FRFs with measured ones in order to identify joint properties has been used in literature. But, it is observed in this study that starting with arbitrary initial values for the unknown joint parameters yield results which are valid only at the frequency at which identification is made. However, using the joint parameters identified by the FRF coupling method suggested in this work as initial estimates in optimization yield improved constant joint parameters (named as updated joint parameters) which can be used at any frequency.

It is experimentally demonstrated that the regenerated FRFs calculated using the updated joint parameters match with the experimentally measured ones perfectly at all frequencies. After presenting a simulation study to show the application of the method, two experimental case studies are presented to verify the method suggested in this work and to illustrate the accuracy and applicability of the method in real life applications. In the first experimental case study, by using the identified bolted joint parameters the FRFs of the coupled structure are calculated at a point which is not used in the identification of joint properties and these values are found to be perfectly matching with the measured FRFs. In the second experimental case study, the identified joint parameters are used in calculating the FRFs of a new assembly constructed with the same bolt but with a beam longer than the one used in identification. The predicted FRFs are compared with experimentally measured ones and once again a very good match is observed. Thus, it is concluded in this work that joint identification method proposed is practical to apply, yields very accurate results, at least for bolted beam joints, and promising to be applied to more complex structural systems.

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